Catalan Conjecture Solution

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A. $X^p - Y^q$ all are integers, $Y>X>1$, $(X,Y)=1$, $p>q>1$, $(p,q)=1$. .... (A)

B. Change the above form (A) to :

$[x(1-k)]^p - [x(1+k)]^q = a^p - (a+c)^q$. ....(B)

here $p>q>1$, $1>k>0$, set $k= c/ 2a+c$, $x=2a+c /2$, a and c are integers, we have to have $a>1$ and $c>0$.

Then we can get any "a" and "c" you want,and keep the meaning needed.

C. 1. We want $\{[x(1-k)]^p - [x(1+k)]^q\}^2 = 1$, we get:

$[x(1-k)]^{2p} + [x(1+k)]^{2q} - 2 x^{(p+q)} (1-k)^p (1+k)^q = 1$.

2.Suppose we can find a "k" that can make:

$[x(1-k)]^{2p} + [x(1+k)]^{2q}$ as small as possible, and make $- 2 x^{(p+q)} (1-k)^p (1+k)^q$ as big as possible,

then we can judge the whole question.

3. First,we ask for the extreme of $z = (1-k)^p (1+k)^q$.

Put $z' = dz/dk = 0$, note $z'' < 0$, we get $k = -(p-q) / p+q$.

Here $k<0$, because $p>q$, but it should be $0<k<1$.

4. Never mind, we just take $k= +(p-q) / p+q$,

why can we do so? There's a simple reason:

(It's not difficult to prove,you can do it by yourself)

when 0<x<1-s, 0<s< 1/2, 's' depends on p and q,
for any x, k= -(p-q) / p+q makes the absolute value
\ [x(1-k)]^p - [x(1+k)]^q \ smaller
than k= p-q /p+q does.

On the contrary, when x>1 or =1, it is k= p-q /p+q,
that makes the absolute value \ \ smaller than
k = - (p-q) / p+q does.

D. 1. Anyway we take k = p-q /p+q that is from the extreme proceeding, but we still don't know whether it is the extreme point for the above expression (B).

2. Let's put it into (B) to see what will happen, and let k decide x, k= p-q / p+q = c/ 2a+c, x= 2a+c /2,
just let c= p-q and a= q, we get:
[x(1-k)]^p - [x(1+x)]^q = q^p - p^q.
If we want the absolute value \ q^p - p^q \ = 1,
it's easy to prove that it should be p - q = 1 to guarantee the possible smallest.

We might as well suppose q^p - p^q = 0, thus p = q.
Now we want \ q^p - p^q \ = 0 + 1, the smallest difference,and note p > q, thus p - q = 1 is necessary.

This is the simplest expression concerning the extreme value, and all are the basic integers.

3. The expression \ q^p - p^q \ = 1 with p - q = 1 shows us the "path" to look for the smallest value of \ q^p - p^q \, if there is such \ \.

4. And it is the only path we can and should follow,because it is from the extreme process.

5. Yes,there are other forms to make the value \ \ ups and downs, but definitely cannot influence us to reach the goal through the main path that is from extreme process. In other words, that other forms only can approach the goal as much as possible,but the final result is not the smallest.

6. Back to \ q^p - p^q \ with p - q= 1,
it is obvious that the value of \ \ is divergent, getting bigger and bigger with p and q becoming bigger.

7. So we just need to try some small ps and qs to test our thinking.
The bottom and the smallest is \ 2^3 - 3^2 \ = 1,
here p = 3, q = 2 and p - q = 1, and it is only one.
And never can be repeated again.

E. Back to the above B.
if we take a= 2, c= 1, the smallest integer we can take,then
k = c/ 2a+c = 1/5, and x = 5/2,
this is exactly k = p-q / p+q = 1/5, and x = 5/2,
when q = 2, p =3, also the smallest one we can take.
This is the real bottom one.

F. \2^3 - 3^2\ = 1 is the smallest and only one
in our universe!

Theorem lkx.
For Catalan's X^p - Y^q. Y > X > 1, (X,Y) = 1.

p > q > 1, (p,q) = 1.
All are integers.

We have the integer extreme value form:

absolute value \ q^p - p^q \ Here p - q = 1.

the smallest and only one is:

\ 2^3 - 3^2 \ = 1.

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