Dear friend,

About Catalan's conjecture,

1. If we still depend on so called 'Double Wieferich Primes', depend on computation, the problem has already gone to dead end. Why?
Because we can never do it completely. Maybe it is God's job.

2. Moreover, that so called 'Criterion' cannot include (p, q) = (3, 2),
that is to say, that the only result | 2^3 – 3^2 | = 1 exists is an accidental phenomenon, just happening to be so. How weird it is !
How can we imagine that something is 'living' without 'roots' ?!
It's amazing that we were so confused with something wrong for so long time.

In fact, that the only result | 2^3 – 3^2 | = 1 comes from extreme process, it is inevitable without any doubt. My paper says it very clearly.

Now I send you 'My paper + The geometry + The supplement', total 7 pages in PDF [Binder 1] attached here, very concise, hope you get interested.

Yes, I can also make some small mistakes here or there.
But, in general terms, I can definitely say I have done it, and clearly and thoroughly, and should let the whole world know it, it's my duty.

Any questions, criticism and information are always welcome.

Best regards

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The Extreme Solution to Catalan Conjecture

Author Li Kexiong

A. Catalan's $X^p - Y^q$ $Y > X > 1$, $(X, Y) = 1$ $p > q > 1$, $(p, q) = 1$.
All the numbers are integers. ……………………………………………….. (A)

B. Change the above (A) into:
$[x(1-k)]^p - [x(1+k)]^q = a^p - (a+c)^q$ ………………………………………………….. (B)
Here we set $x= 2a+c /2$, $k= c/ 2a+c$. a and c are integers, $a>1$, $c>0$, $(a, c)= 1$.
Thus we can get any 'a' and 'c' you want, and keep the meaning needed.
Note, here $0 < k = c/ 2a+c < 1$.

C. We want $\{[x(1-k)]^p - [x(1+k)]^q\}^2 = 1$,
we get $[x(1-k)]^{(2p)} + [x(1+k)]^{(2q)} - 2x^{(p+q)} (1-k)^p (1+k)^q = 1$ ……………………… (C)
the smallest difference.

D. Now suppose we can find a 'k' which can make $[x(1-k)]^{(2p)} + [x(1+k)]^{(2q)}$ as small as possible, and make $-2 x^{(p+q)} (1-k)^p (1+k)^q$ as big as possible, then we can judge the whole question.

First, we ask for the extreme of $Z= (1-k)^p (1+k)^q$, put $Z' = dZ/dk =0$, note $Z''<0$, and $p>q>1$
we get $k =- (p-q) / (p+q)$, here $k<0$, but it should be $0<k<1$, according to the above B.

E. Never mind, we just take $k=+(p-q)/(p+q)$, why can we do so?
There is a simple reason:

1. When $0<x<1-s$, here $0<s< 1/2$, 's' depends on p and q, $k=- (p-q)/(p+q)$ makes the absolute value $\backslash[x(1-k)]^p - [x(1+k)]^q \backslash$ smaller than $k=+(p-q)/(p+q)$ does.
2. On the contrary, when $x>1$ or $=1$, it is $k=+(p-q)/(p+q)$ that makes the absolute value $\backslash [x(1-k)]^p - [x(1+k)]^q \backslash$ smaller than $k=-(p-q)/(p+q)$ does.

3. How to get the above two points:
For the clearness $k=-(p-q)/(p+q)$, $j=-k=+(p-q)/(p+q)$ put them into B, we get

$$K= [x(1-k)]^p - [x(1+k)]^q.$$
$$J=[x(1-j)]^p - [x(1+j)]^q.$$

Then we discuss $K^2 - J^2 = (K - J) (K+ J) < 0$, or > 0, or $= 0$,
because $p>q >1$, $(K - J)$ is always > 0, so we mainly discuss $(K+ J) < 0$, or > 0, or $= 0$.
when $(K + J) = 0$, there is a "turning point" T:
$x = T = \{ [(p^q + q^q) / (p^p + q^p)]^{(1 / p-q)} \} (p+q /2)$. Note, $1/2 < T < 1$,
When $x < T$, the above Point 1. is effective； $T = 1-s$ in the above E.1.
$x > T$, Point2. is effective;
$x = T$, get two equal absolute values.

F. From E.2, we know that we are only interested in k = +(p-q)/(p+q) when x>1.
Can we say it is also an extreme point? The answer is "Yes" without any doubt.

1. **From the above (c), we can - 2 x^(p+q) (1-k)^p (1+k)^q**
   = - 2 x^(p+q) (1-k / 1+k)^(p-q) (1-k)^q (1+k)^p.
2. **W = (1-k)^q (1+k)^p, ask for the extreme of W, put W' = dW/dk = 0,**
   we get k = +(p-q) / (p+q), note W" also < 0 at k = +(p-q) / (p+q),
   this k is exactly what we want, not only it is conformed with E.2, but also
   it is the extreme point of W .

3. Please compare W = (1-k)^q (1+k)^p with Z = (1-k)^p (1+k)^q,
their extreme points are the opposite numbers, but the both get the same
extreme value, they are actually two sides of the same coin.

4. When we get W, we also get V = (1-k / 1+k)^(p-q) from the above F.1.
Since W's extreme point 0< k=+(p-q)/(p+q) <1, V will become the biggest of
itself when p - q = 1. Actually there is no contradiction between W and V,
later we'll mention it again.

G. 1. Now that we get k=+(p-q)/(p+q) as the extreme point of W, let's put this k
into { [x(1-k)]^p – [x(1+k)]^q }^2 = [x(1-k)]^(2p) + [x(1+k)]^(2q) – 2 x^(p+q) V W
= (q^p – p^q)^2. Note, now x is already > 1,
when we use k=+(p-q)/(p+q).
here let k=+(p-q)/(p+q) = c/ 2a+c, just let c=p-q, a=q, then get x= 2a+c /2 = p+q /2.
Totally conformed with (B)'s requirement.

2. Here V= (1-k / 1+k)^(p-q) = (q/p)^(p-q) should be the biggest of itself according to
the demand of extreme process, thus p-q=1 is necessary.

3. **When p-q=1 makes V= (q/p)^(p-q) = (p-1)/p the biggest it can be,**
   x^(p+q) W = (p+q /2)^(p+q) (2/p+q)^(p+q) [q^q] [p^p] = q^q p^p = (p-1)^(p-1) p^p.
   no doubt, it is the biggest and only result that we can get. Why?
   The reason is quite simple : because (p, q) = 1, so (pq, p+q) = 1,
   so [q^q p^p, (p+q)^(p+q)] = 1;
   and, only x=(p+q)/2 = [p+(p-1)]/2 = (2p-1)/2 can make x^(p+q) W to become the integer,
   x=(p+q)/2 cannot be more or less, otherwise meaningless.

4. **Now -2 x^(p+q) W V = -2 (p-1)^(p-1) p^p (p-1)/p = -2 (p-1)^p p^(p-1).**
   No doubt, it is the extreme value and no contradiction between x^(p+q) W and V.

5. **k=+(p-q)/(p+q) really makes the extreme value at last under the condition p-q=1.**
   From now on, p –q =1 has already become the necessary condition together with
   (q^p – p^q)^2.

H. 1. Or we can put k=+(p-q)/(p+q) and x= (p+q)/2 into (B) directly, get the absolute value \ [x(1-k)]^p – [x(1+k)]^q \ = \ q^p – p^q \.

2.Definitely we want \ q^p – p^q \ = 1 started from the beginning (C).
It's easy to prove that it should be still p-q=1 to guarantee the possible smallest.
(We might as well suppose q^p – p^q = 0, thus p=q,
now p>q>1, we want absolute value \ q^p – p^q \ = 0+1, the smallest difference,
thus p – q = 1 is necessary.)

3. When W gets its extreme value with k=+(p-q)/(p+q) and p-q=1, V also becomes the biggest of itself because of p-q=1, W and V are conformed with each other, nothing contradicted each other. This shows our way in F.1 is all right.

I. \ q^p – p^q \ with p-q=1 is the essential necessary condition for us to look for the the smallest \ X^p – Y^q \ , this is the way we can and should follow because it is from the extreme process. Actually, Catalan conjecture is an extreme problem concerning integers.

J. It's obvious that the value of \ q^p – p^q \ with p – q = 1 is divergent,
getting bigger and bigger with p and q becoming bigger.
So we just need to try some small ps and qs to test our thinking.
The smallest and only one is : \ 2^3 – 3^2 \ = 1 and never can be repeated again.

Theorem Lkx
For Catalan's X^p – Y^q Y > X > 1 (X, Y) = 1, p > q > 1 (p, q) = 1,
all are integers.
We have the integer extreme form (the essential necessary condition) :
absolute value \ q^p – p^q \ with p – q = 1.
The smallest and only one is \ 2^3 – 3^2 \ = 1.

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The Plane Geometry Explanation for the paper

‘ The Extreme Solution to Catalan Conjecture’

By Kexiong Li

Please prepare a compass and a ruler if need.
Please note, always clockwise. Now let’s begin:

I. Make a right-angled triangle ABC:
1. Suppose that AB = [x(1-k)]^p. BC = [x(1+k)]^q. x > 1, 0 < k < 1.
AB < BC, just for the convenience.
2. AC is the hypotenuse and is level.
3. The point B is the vertex, the angle ABC = 90 degrees.

II. On the hypotenuse AC, we can make a square AA’C’C, A’ is above A, C’ is above C.

III. On each side of the square AA’C’C, we can make a half-circle inside the square,
arc AC, arc A’A, arc C’A’ and arcCC’,
the diameter = the side length of the square.
The 4 arcs meet together at the center point of the square, the point o.

IIII. The arcAC surely meets the vertex B;
Prolong AB to meet the arcA’A at the point B1;
Prolong A’B1 to meet the arcC’A’ at the point B2;
Prolong C’B2 to meet the arcCC’ at the point B3, the point B3 is surely on BC.
Thus the line segments BB1, B1B2, B2B3 and B3B have formed a smaller square BB1B2B3 inside the bigger squareAA’C’C.
The both squares, the bigger and the smaller, have the same center, the point o.

V. At the same time, we get 4 right-angled triangles ABC, A’B1A, C’B2A’ and CB3C’,
the 4 are exactly the same triangles.

VI. Now we get:
The area of the bigger squareAA’C’C = AC^2 = [x(1-k)]^(2p) + [x(1+k)]^(2q);
The area of the 4 right-angled triangles = 4/2 AB BC = 2 x^(p+q) (1-k)^p (1+k)^q;
And we want the area of the smaller square BB1B2B3
= [x(1-k)]^(2p) + [x(1+k)]^(2q) – 2 x^(p+q) (1-k)^p (1+k)^q
to become the smallest in general,
and then, if possible, to become 1, the smallest integer > 0.

VII. **According to the picture we have made above,**

If we want

$(X^p - Y^q)^2 = \{ [x(1-k)]^p - [x(1+k)]^q \}^2 = AC^2 - 2\,AB\,BC$

= the area of the squareBB1B2B3 = 1,

from integer point of view, it is the smallest result.

Integers are a part of real numbers.

Therefore, whenever the area of the 4 right-angled triangles should be the biggest, it becomes the precondition for the whole problem, it is absolutely necessary.

VIII. **We do know that the area of the 4 right-angled triangles =** $2\,x^{(p+q)}\,(1-k)^p\,(1+k)^q$ **can be made as big as possible. Why?**

Because 1. **x is any** $x>1$**, no matter how x is big or small, x will not influence k, x can be separated from k for the time being.**

2. **We can make** $(1-k)^p\,(1+k)^q$ **as big as possible till it can not be bigger any more,till its biggest, till its extreme value.**

3. **Thus we found the extreme point** $k = +(p-q)/(p+q)$**, when** $x > 1$**.**
 Please note, when $p-q = 1$**, we can get the only extreme value, the real biggest. It is bigger than any one from** $p-q > 1$**.**

IX. **Then we have the possibility to think about whether the expression** $[x(1-k)]^{(2p)} + [x(1+k)]^{(2q)} - 2\,x^{(p+q)}\,(1-k)^p\,(1+k)^q$ **can be turned into an integer expression and get the smallest integer result.**

If it can be, it will definitely include $\backslash\, 2^3 - 3^2 \,\backslash = 1$**.**

X. **Every step is 'we have to', maybe it's the only way out, and we finally got the right answer.**

XI. **The other way round: We can fix AC first, different from the above, but the final result will be the same.**

1. **Begin with any line segment** $AC = u > 0$ **as 'the base'.**
 We can have the half-circle arcAC on AC.

2. **Put** $AC^2 = u^2 = [x(1-k)]^{(2p)} + [x(1+k)]^{(2q)}$**, please treat it as an equation. …(1)**
 $k = +(p-q)/(p+q)$ **is the extreme point of** $(1-k)^p\,(1+k)^q$ **when** $x > 1$**.**
 Especially, we can obtain the real extreme value when $p-q=1$**.**

3. **Now, only 'x' is unknown in the equation (1).**
 Theoretically, we can solve the equation, obtain the root $x = v$**.**
 And, we can always say $x = v > 1$**, except** $AC = u = 0$**.**

4. **Then,** $AB = [v(1-k)]^p$ $BC = [v(1+k)]^q$**, AB and BC form a right-angled triangle.**
 The arcAC meets the vertex B.

5. Then we get the biggest right-angled triangle ABC within the half-circle arcAC.
   So the possible smallest result = AC^2 - 2 AB BC.

6. Please remember AC is an any line-segment > 0.
   So our proof has the general validity.

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3 Supplemental Proofs for the Paper

‘ The Extreme Solution to Catalan Conjecture’

By Kexiong Li

I. In E3, the turning point T = {[(p^q +q^q)/(p^p+q^p)]^(1/p-q)} (p+q /2).
1/2 < T < 1.

Proof.

When p – q = 1, T = 1 – [p^(p-1) – (p-1)^(p-1)] / 2[p^p + (p-1)^p] < 1.
When p >q and q = 2, T > {[p^p + p^(p-2) 2^2] / (p^p + 2^p)}^(1/p-2) 1/2 > 1/2.

II. In F2, W’ = dW/ dk = 0, k = +(p-q) / (p+q).
W” contains - pq < 0, at this k.

III. In G4 and H3, when k = +(p-q) / (p+q) and p-q = 1, the function (1-k)^p (1+k)^q
can obtain the biggest extreme value, bigger than any one from p-q=n >1.

Proof.

Because (2p)^(n-1) (2p-1) / (2p-n)^n > [(2p-1)^2 (p-n) / (2p-n)^2 (p-1)]^p, n > 1,
the left side > 1 > the right side --- the numerator < the denominator in the [ ].
so the extreme value:
[2 / (2p-1)]^(2p-1) p^(p-1) (p-1)^p > [2 / (2p-n)]^(2p-n) p^(p-n) (p-n)^p.

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