The Solution of the Open Problem 3X+1

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A. Let's see an example first, it can save many words for the definitions,(to be easier, X is always an odd number)

X=15

3×15+1=46
46/2=23 we call it "enhancing step".

23×3+1=70
70/2=35

35×3+1=106
106/2=53

53×3+1=160
160/2=80

The above are 4 enhancing steps --- 4-time continuous enhancing steps.
80-40-20-10-5 then, 4 "reducing steps" --- 4-time continuous reducing steps.

5×3+1=16
16-8-4-2-1

1×3+1=4
4-2-1 "recycle round".

B. The reducing speed is 2 for each reducing step.
How about the enhancing speed? We will discuss it later.

C. Theorem kx 1.
In the 3X+1 proceeding, if X+1=2^n ×y, here y is an odd number, the proceeding gets continuous n steps of enhancing immediately.

Proof.-- simplified.
According to the 3X+1 proceeding, if X+1 = 2^n y, the basic fact is
3X+1 / 2 = 3^1 X + 3^1 - 2^1 / 2^1. ..... step 1.
Then 3^2 X + 3^2 - 2^2 / 2^2 . ... step 2.
...
3^k X + 3^k - 2^k / 2^k. ...... step k.
...
note, here all the X are from the original 3X+1. n > k.

It's obvious that $X+1 = 2^n y$, $2^n > 2^k$, the result of step k is still an odd number, it maens we have to go on enhancing steps.
If finally $k = n$, we will get $3^n y - 1$ = an even number, it means that after n steps of continuous enhancing, now the reduding step(or steps) begins, note y is an odd number.

Please check it with the beginning example, X=15.

D. Theorem kx 2.
Enhancing numbers that can make enhancing is as many as reducing numbers exactly. They are paired odd+1=even, the same n times.

Proof. Because they are matched to pairs.
If the odd X produces n-times enhacing, then the even
X+1 produces n-times reducing.

E. Theorem kx 3.
The probability of the number of continuous enhancing or reducing n-times is $1/2^{(n+1)}$ in the whole natural numbers. Approximately showing within finite numbers, the more numbers, the more accurate.

Proof. Everyone can do it, just look at the basic facts.
$1/2^2 + 1/2^3 + ... + 1/2^k = 1/2(1- 1/2^k)$ total enhancing pb +total reducing pb= $1- 1/2^k$
enhancing probability = reducing probability 2^k means almost the biggest one after enhancing.(2^k =< the biggest).If k is unlimited, the result is 1.

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F. Now let's set up a "model":(KX model)

1.Suppose we have 2n same shape balls, n white balls and n red ones.

2.According to the ratio above mentioned, to mark the balls,

for instance, take 1/2 red ones to mark "1" on each of them, then
take 1/4 red ones to mark 2.............
1/8 3
just each time take the half.
$1/2^k$ .... k
....
till the red ones are finished.
The same way to mark the white.

3.After mixing the two parts very well, randomly pick them out one by one.

4.Suppose after 2k times, we add up all the numbers marked on the red ones; the same way for the white ones.
Let's check the two sums to see what will happen?
This is "the essence of 3X+1".

G. Theorem kx 6.
The random picking proceeding, according to the kx-model, will always reflex a real counting proceeding of 3X+1 of a certain natural number, despite the steps more or less.
Proof. the whole natural numbers contain every possibility.
we don't know where the certain one is, but it exists somewhere, waiting for your random picking steps.

H. Theorem kx 4.
Randomly, if talk about whole natural numbers, the reducing probabilities is the same as enhancing.

Proof. Just like picking up an even number or an odd one each time randomly.

I. Now we have to talk about the speed, because our purpose is discussing the convergence or divergence of 3X+1.
The reducing speed of each reducing step is 2.
How about the enhancing speed of each enhancing step?

1. Suppose X+1=2^n ×y, x and y are odd numbers.
According to the above Theorem kx 1. 3X+1 gets n-time continuous enhancing steps immediately:
3X+1 /2 =(3^1×X +3^1 -2^1)/2^1. the enhancing step 1.
Then,(3^2×X +3^2 -2^2)/2^2. ....step 2.
...
[3^(n-1)×X +3^(n-1) -2^(n-1)]/2^(n-1). ...step n-1. finally,(3^n×X +3^n -2^n)/2^n. ...... step n.
Please note, here all the X are the original X from 3X+1.

2. Step n / Step(n-1),after this division, We get,

Theorem kx(special).
In the 3X+1 counting proceeding, when X+1= 2^n×y,y is odd, the enhancing speed of each enhancing step is

2 > 1.6 > or = 3/2 + 1/[3^(k-1)×y×2^(n-k+2) -2] > 3/2
k means step k. from 1. to n.

3.The enhancing speed is a changing one, not a constant, it depends on n and y. The bigger n and the bigger X, the less enhancing speed. It will go to 3/2 at last.

4.We can take y=3 and n=1,it means X=5, we get the quickest enhancing speed 3/2 + 1/10 = 16/10 =1.6, far less than the reducing speed 2.

I. Theorem kx 5.
The final result of 3X+1 proceeding will not be influenced by the proceeding order decided by X. (the order seems random and it is really random due to different random numbers.)
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Proof. Suppose the original 3X+1/2=a, now we can use the quickest enhancing speed for every enhancing step according to the special theorem kx, we try our best to enhance the value, roughly, a×(1.6)^k /2^m, k times enhancing and m times reducing. Where is the timing order? According to the above theorems, k almost equals to m, the bigger the number, the more obvious.

J. Theorem kx 7.
The proceeding of 3X+1 is convergent at last.

Proof. 1.For the randomness, we have the "opportunity equality" from the nature of natural numbers;

2.Because of the speed 1.6 < 2,the reducing force is stronger than the enhancing force.

3.The above two points have a common idea:
The bigger X, the more steps and more numbers involved, the more obvious the convergent result of 3X+1.

so we just need to test small successive numbers, for instance 3,5,7,9,...101, if they do, needless to say bigger ones.

K. Theorem kx 8.
The proceeding of 3X+1 only can get a finite "recycle round" at last.
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Proof. 1. the proceeding of 3X+1 is convergent, the value becoming smaller and smaller, but the value is always an integer.
2. the procedure can be going on forever...

Where is the way out? only a finite recycle round, that is the reasonable thing, just as real practice shows.

L. Would you like to try: 5X+1 or 7X+1?
other conditions are the same as 3X+1.

Would you like to discuss the general form aX+b/c ?

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